

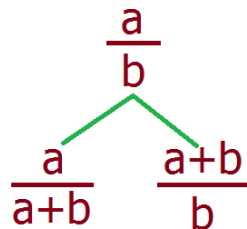
## 70. Little Bo Peep counts her sheep...

Little Bo Peep's business was booming. Sheep were in demand, and when it came to shepherdesses, there were none finer. She had, in fact, now been successful enough to collect an infinite number of sheep in her flock. Which in turn gave her a headache; how to count them? She had tried putting 1, 2, 3, 4... onto the side of each sheep in turn, and after a long while, during which she had become infinitely quick at painting a number onto the flank of a sheep, she had managed to number them all. This did not, however, completely answer her problem. 'Bo Peep,' said a local farmer, exasperated, 'We paint 1, 2, 3, 4... onto the sides of OUR sheep. How can we tell yours and ours apart?' Bo Peep's forehead creased as she grappled with this conundrum. Then it came to her in a flash; 'I'll paint fractions onto the sides of my sheep!'

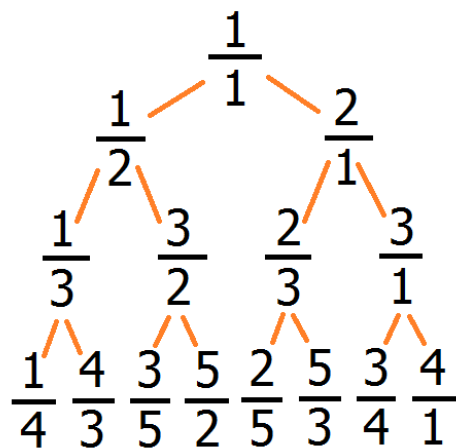
The next day, she set to work. 'Every sheep must have a fraction – and every fraction must correspond to a sheep', she thought. 'And I will count  $1/1$ ,  $2/2$ ,  $3/3$  as the same fraction – equivalent fractions must only occur once, in their simplest form!' she cried. 'And in addition, I must be able to put my sheep in order.' Immediately she

was struck with a dilemma – would she try to arrange her sheep in order of size? ‘Between any two sheep  $a/b$  and  $c/d$  comes the sheep  $(a+c)/(b+d)$ ,’ she realised, despairingly, ‘So I can never put them all in order of size.’

She went for a hike to clear her head. As she looked down onto her flock from the top of a nearby fell, it came to her in suddenly. ‘A fraction can give birth to two others, like this!’ she cried.



‘Now if I start with  $1/1$ , I get this.’



‘Every fraction occurs just once in its simplest form. Now if I put the rows together in a long line, I have ordered my sheep!’

That afternoon, she set to with a paint brush. Two hours later, a passing shepherd found her, fast asleep...

*With thanks to Calkin and Wilf at*

*<http://www.math.upenn.edu/~wilf/website/recounting.pdf>*

Jonny Griffiths, [jonny.griffiths@ntlworld.com](mailto:jonny.griffiths@ntlworld.com), Sept 2012