## MSV 27: Answers


$A$ and $B$ are independent events if and only if $P(A \mid B)=P(A)$.
So from the diagram, $A$ and $B$ are independent events if and only if

$$
\frac{b}{b+c}=a+b=\frac{a+b}{a+b+c+d} .
$$

This is true (multiplying out) if and only if $\mathbf{a c}=\mathbf{b d}$.
We can now note that $\mathbf{0 . 4 5} \times \mathbf{0 . 1}=\mathbf{0 . 3} \times \mathbf{0 . 1 5}$.
So we can assign the four numbers to $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and d in eight different ways that make sense.

| $\mathbf{a}$ | $\mathbf{c}$ | $\mathbf{b}$ | $\mathbf{d}$ |
| :---: | :---: | :---: | :---: |
| 0.45 | 0.1 | 0.3 | 0.15 |
| 0.45 | 0.1 | 0.15 | 0.3 |
| 0.1 | 0.45 | 0.3 | 0.15 |
| 0.1 | 0.45 | 0.15 | 0.3 |
| 0.3 | 0.15 | 0.45 | 0.1 |
| 0.3 | 0.15 | 0.1 | 0.45 |
| 0.15 | 0.3 | 0.45 | 0.1 |
| 0.15 | 0.3 | 0.1 | 0.45 |

There are 24 ways to allocate the four numbers altogether,
So $\mathrm{P}(\mathrm{A}$ and B are independent) is $\mathbf{1 / 3}$.

We could look at this problem this way:
If $A$ and $B$ are independent, then so are $A^{\prime}$ and $B$, and $A$ and $B^{\prime}$, and $A^{\prime}$ and $B^{\prime}$.
The Venn diagram for each of these eight pairs of events is below.

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