Mini-Theorems

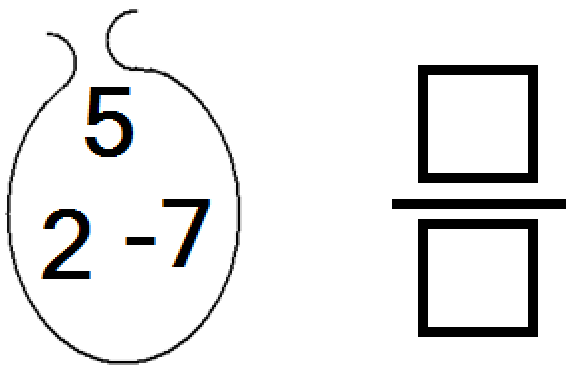
So we have a skill we would like our students to practise. Option One; set a back-breaking set of exercises to practise the required skill. Option Two; set an extended slow-burn exercise, generated by the student, that practises the required skill en passant and that works towards a 'mini-theorem'. The idea here is to bring off a magic trick of sorts.

Let me give you an example based around fractions. The exercise goes like this:

Pick three different non-zero whole numbers that add to 0,

and put them into a bag.

Extra rule: no pair of your numbers should have a common factor.



Now pick two numbers from the bag (no repeats!)

and put them into the squares to make a fraction.

How many different fractions can you make? Write them down.

Put the fractions you have into order.

What do we get if we multiply all the fractions?

What do we get if we add all the fractions?

Compare notes on this with your colleagues.

Can you prove this will always work?

Note that the student starts by choosing their numbers – they create the question, and so they have a vested interest in making it work. The exercise is differentiated – not everybody will make it to the final line, but you hope that everyone will be able to create their starting fractions, and that most will be able to order them.

How might a bright student do the last part? Say the bag contains a, b, c where a + b + c = 0. We can make 6 different fractions, $\frac{a}{b}$, $\frac{c}{b}$, $\frac{a}{c}$, $\frac{b}{c}$, $\frac{b}{a}$ and $\frac{c}{a}$.

Adding them gives

$$\frac{a+c}{b}+\frac{a+b}{c}+\frac{b+c}{a}$$

$$= \frac{-b}{b}+\frac{-c}{c}+\frac{-a}{a}$$

$$= -3.$$

If you wonder how this exercise was constructed, I think you can see here that the end of the very first line of the problem, 'that add to 0', was in fact the very last part of the problem to be written.

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